Subject: Re: determining if a point is "inside" or "outside" a shape
Posted by Job von Rango on Thu, 21 Oct 1999 07:00:00 GMT
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Ther is another analytic way to solve the problem:

How to determine wether a point lies inside an arbitrary 2-dimensional polygon or not?

IDEA:
Look at the sum of all angles tended between all pairs of neighboured vertics the polygon and the given point.

IN DETAIL:
Assume we have the n vertices of polgons ordered at the positions:
v_1, v_2, ... ,v_n
The given point is denoted by p.

Now connect the given point p with all n neighboured vertices, and get the n vectors beginning in p and ending in the vertices:

```
vec_1 = vector(p_0, v_1)
...
vec_n = vector(p_0, v_n)
```

Now add all n angles tended between the 2 vectors vec_i and vec_(i+1):

```
angle_i = angle(vec_i,vec_(i+1))
```

and calculate the sum:

```
             n
anglesum = sum  angle_i
            i=1
```

(Use vec_1 again for vec_(n+1) in the last angle_n)

RESULT:

```
            _
           |  2*pi       inside
anglesum = -|           if p is       the polygon
           |_   0        outside
```

IMPORTANT:
Take into account the correct sign of the angles

and use the vertices in ascending order!
Use e.g. the vector crossproduct:

vec_i x vec_(i+1)

in order to retrieve the absolut value |...| for the angle:

|angle_i| = inv sin(|vec_i x vec_(i+1)|)

The valid sign for angle_i is given by looking at the scalar product
of the crossproduct from above and a fixed vector vec_perp, perpendicular
to the area of the polygon:

```
               vec_perp * (  vec_i x vec_(i+1)  )
sign_i =   ------------------------------------
             | vec_perp * (  vec_i x vec_(i+1)  ) |
```

Now we get the correct value for all angles:

angle_i = sign_i * |angle_i|

REMARK:
Examine the case of a given point inside the polygon, but very near
to the edge between two vertices v_i and v_(i+1). The contributing
angle will be:

angle_i = + pi - epsilon

(where epsilon denotes a small positive angle.)
If we shift p over the edge to the outside of the polygon
(but very near again to the edge)
the contributing angle will be:

angle_i = - pi + epsilon'

The switch of the sign is the reason for the discontinuity (2*pi <-> 0)
of anglesum for points moving from inside to the outside of the polygon!

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Job v. Rango   Medizinische Klinik I
Dipl.-Phys.   D-52074 Aachen
Pauwelsstrasse 30
Tel. : 049 / (0)241 / 80-89832

Fax : 049 / (0)241 / 88-88414
Email: jran@pcserver.mk1.rwth-aachen.de