Subject: Re: FFT example. Help!
Posted by asb on Tue, 02 May 2000 07:00:00 GMT
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I think your confusion stems from a misunderstanding of what the FFT does. There are at least three related, but not identical, operations that are referred to as "Fourier transforms".

1) Fourier integral.
A function f(x) that is "well behaved", _continuous_ and _not periodic_ has a Fourier transform F(k) that is "well behaved", continuous and not periodic. F and f are related by the Fourier transform and Fourier inversion formulas:
F(k)=int(f(x) exp(-i kx) dx)
and
f(x)=int(F(k) exp(i kx) dx)
The limits of the integral are + and - infinity.

2) Fourier series.
If the function f(x) is a periodic function of x with period L, its Fourier transform is zero for all k that are not integral multiples of 2 Pi /L, and the integral diverges if k is an integral multiple of 2 Pi / L. F(k) can be expressed as a sum of dirac delta functions, but it is more convenient to simplify the notation and write f(x) as a Fourier series:
f(x) = sum(An sin(n Pi x / L) + Bn cos(n Pi x / L))
where the Fourier coefficients An and Bn are
An=(2/L) int(f(x) sin(n pi x / L) dx)
Bn=(2/L) int(f(x) cos(n pi x / L) dx)

3) Discrete Fourier transform (DFT)
If the function F(x) is a periodic function of x with period L and is only defined at N equally spaced discrete points xn = n L / N, n = 0, N-1, then its Fourier transform is a periodic function of k = 2 Pi / L and is nonzero only at the discrete values km = 2 m Pi / L, m = 0, N-1. Instead of considering the functions F(x) and f(k), which are sums of delta function, it is easier to consider Fm and fn, the coefficients of the delta functions. These coefficients are related by the discrete fourier tranform:

Fm = sum(fn exp(-i 2 Pi m n / N))

and its inverse

fn = (1/N) sum(Fm exp(i 2 Pi m n / N))

The FFT is a fast algorithm for computing the discrete fourier transform.

If you want an example using the FFT that "makes sense" (I asume that you mean one that you can compute analytically), you have to remember that the FFT computes the discrete Fourier transform, not the Fourier integral.
The simplest example is
fn = 1 for all n
Fm = N for m = 0; 0 otherwise

The FFT is frequently applied to the problem of estimating the Fourier integral of a non-periodic function whose value is only known at a discrete set of sampled points. One then must cope with errors introduced by sampling (aliasing) and truncation (Gibbs ringing).

For a good overview, look in Numerical Recipes by Press et al.

I hope this helps.