Subject: Re: Meaning of outer product
Posted by James Kuyper on Sat, 13 Jul 2002 18:17:05 GMT
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Paul Sorenson wrote:
>
> Greetings,
>
> IDL documentation says: "Note - If A and B arguments are vectors, then C =
> MATRIX_MULTIPLY(A, B) is a matrix with C_ij = A_iB_j. Mathematically, this
> is equivalent to the outer product. . . ."  But I'm having difficulty
> reconciling this with my understanding of outer product. . .
>
>      c.x = a.y*b.z - a.z*b.y
>      c.y = a.z*b.x - a.x*b.z
>      c.z = a.x*b.y - a.y*b.x
>
> ... which yields a vector (c) instead of a 2D array.  Can anyone shed some
> light on this?

The outer product you are referring to is also called the cross-product, and is different from the outer product that the IDL documentation is talking about. However, the two concepts are related. In general, the outer product of two vectors could be written as:

```
OP = [[a.x*b*x, a.x*b.y, a.x*b.x],
  [a.y*b*x, a.y*b.y, a.y*b.x],
  [a.z*b*x, a.z*b.y, a.z*b.x]]
```

You can create such an outer product by matrix multiplication of column vector by a row vector (matrix multiplication of a row vector by a column vector gives their dot product).

While 'OP' has 9 different elements, they were calculated from only six different original numbers. The entire information content of that array can be summarized by forming two other matrices:

```
 SYM = (OP + transpose(OP))/2 Symmetrized
 ASYM = (OP - transpose(OP)) Anti-symmetrized
```

The ASYM matrix necessarily has 0 on it's diagonal elements, and the off-diagonal elements come in pairs that are exactly equal in size, and opposite in sign. Therefore, it can be described by exactly three independent numbers. The cross-product is a special rearrangement of those three numbers to create a pseudo-vector. It's a pseudo-vector because it can be rotated the same way as a real vector, but remains unchanged by mirror reflections. Mathematically, it's called the "dual" of the anti-symmetrized outer product.

I'm not sure how the cross-produce ended up also being called the "outer product", but it's probably because of this connection. For some purposes, the three numbers in the cross-product represent all of the useful information that's contained in the outer product.