Subject: Re: Are the field lines the trajectory of a particle with mass M?
Posted by jeyadev on Mon, 15 Mar 2004 22:04:33 GMT
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In article <405481EF.6359E0DA@saicmodis.com>,
James Kuyper <kuyper@saicmodis.com> wrote:
> pakachunka wrote:
>>
>> I believe the field lines are not the trajectories... but a friend of
>> mine is driving me crazy, because he says they are.
>>
>> How can I demonstrate that field lines are not the trajectories?
>>
>> I mean: what are field lines, to start?
>
> Your question is too broad. There are many kinds of fields. Field lines

Agreed, ...

> are associated with vector fields. A typical vector field may be
> described by a vector-valued function v(x,y,z,t), which means it has a
> single size and direction for every meaningful combination of x,y,z, and
> t. Field lines are lines associated with a vector field that are
> arranged so that at every point, the tangent to the line at that point
> is in the same direction as the vector field at that point. In other

... but this is a concise answer to the broad question.

> words, the vector field tells the lines where to go.
>
> Now, if the two of your are talking about the velocity field of a fluid,
> then the field lines are indeed exactly the trajectories of the
> individual particles that make up the fluid.

Here, things get complicated. I would suggest a quick read of the first
couple of pages of Chap. 3, 'An Introduction to Fluid Dynamics' by
G. K. Batchelor. There are three kinds of lines you need to worry about.

1. Streamline: A line whose tangent is everywhere parallel to the
velocity vector field v(x,y,z,t) instantaneously. Here, v(x,y,z,t)
is just the velocity of the fluid at point (x,y,z) at time t.

2. The Path Line (or simply Path): This is just the path traced by a
particular element of the fluid. It conincides with the streamline
only when the flow is steady (i.e. the velocity field v does not
depend on time).

3. The Streak Line: The line along which lie all those fluid elements

that earlier passed through a certain point in space. This is the
line seen when one injects smoke into a gas or a dye into a liquid
to get a feel of the 'motion' of the liquid.

> However, fluid dynamics is fairly complicated, and your question gives

Yes! And so, only in the case of the steady state flow do the three
kinds of lines coincide. Thus the velocity field 'lines' are NOT, in
general, "the trajectories of the individual particles that make up the
fluid".

> the impression that you're at a fairly elementary level in physics. In
> that case, the fields you're most likely to run into aren't velocity
> fields, but electrical or gravitational fields. For example, the
> electrical field in the vicinity of a particle with charge Q, at a
> position <x0,y0,z0> has an associated static electrical field at a point
> <x,y,z> which is given by
>
> E(x,y,z) = kQ<x-x0,y-y0,z-z0>/r^2
>
> where r^2=(x-x0)^2+(y-y0)^2+(z-z0)^2.
>
> The important thing is that the electrical field is NOT a velocity
> field, and therefore the field lines are not in general the same as the
> particle trajectories. If the electrical field provides the only force
> that is acting on a particle with a charge of q and a mass of m, then it
> will feel an acceleration of Eq/m. Acceleration is not velocity, it's
> the first derivative of the velocity. There's a connection between the
> electrical field lines and the trajectories of the particals, but it's
> not a simple one.

Paths of charged particles in certain specail configurations of electric
and magentic fields are rather well studied and an undergraduate text in
Classical Mechanincs (e.g. Symon or Marion) as well as books on Classical
Electrodyanmics should have some coverage. These problems are much more
fun when relativity comes in :-)

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