## Subject: Re: Averaging quaternions Posted by Arnold Neumaier on Sun, 21 Mar 2004 17:48:33 GMT View Forum Message <> Reply to Message

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jelansberry wrote:
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- > "Arnold Neumaier" < Arnold. Neumaier@univie.ac.at > wrote in message
- > news:405D56B6.6030403@univie.ac.at...

>> jelansberry wrote:

>>

- >>> I would compute the average of the
- >>> Euler angles, and then convert the resulting average Euler angles back to a
- >>> quaternion (convert the Euler angles to a direction cosine matrix, then
- >>> extract the quaternion).

>>

- >> This has exactly the same problems as averaging over quaternions, since
- >> angles are only unique up to a multiple of pi or 2pi; so the average
- >> depends on whether you represent an angle by a number close to pi or
- >> close to -pi ...

>>

- >> Arnold Neumaier
- >

- "Uniqueness" of the Euler angles is not the issue, it's more an issue of
- > continuity of the angles. Euler angles do not have the "same" problems as
- > averaging over quaternions. My basic beef with averaging quaternions is
- > that the initial result of the average is not a quaternion (i.e., the result
- > does not have unit norm). Euler angles do not suffer from such a
- > complication.

The real part of a unit quaternion (with nonnegative real part) is redundant in that it can be recomputed from the imaginary part. Thus averaging the imaginary parts and recomputing the real part would be a simpler recipe of the same kind as yours with Euler angles. And it would have exactly the same problems as the avarage-and-scale method, although there are no asingularities. It is a matter of non-uniqueness in both cases, which implies that one must make ad hoc normalizations: A choice of sign in the quaternion case, and a choice of some normalization interval in the Euler case. This cannot be done without introducing discontinuities - these are not present in the mathematics but only in the normalization chosen.

- > If all the OP is doing is trying to find the average attitude over some
- > fairly small period of time, then one might expect the Euler angles
- > corresponding to the quaternion samples to fairly continuous.

Not if one of the angle is just a little less than pi and increasing

beyond pi (suddenly becoming -pi)

- > I agree (and my post gave fair warning) that with Euler angles one has to be
- > careful of choosing sequences near the singularity of the sequence.

AND near the normalization bounds! The average-and-scale technique is thus even better since it has no singularities and only the problem with possible discontinuities in the representation.

Arnold	Neumaier
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