Subject: Re: equally spaced points on a hypersphere?
Posted by James Kuyper on Fri, 29 Oct 2004 16:52:08 GMT
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Craig Markwardt wrote:
> Matt Feinstein <nospam@here.com> writes:
>
>> On 29 Oct 2004 07:51:58 -0700, robert.dimeo@nist.gov (Rob Dimeo)
>> wrote:
>>
>>
>>> Hi,
>>>
>>> I would like to create (n+1) equidistant points on an n-dimensional
>>> sphere. The initial information provided is the center of the sphere,
>>> the radius, and *any* point on the sphere. From that you need to find
>>> the coordinates for the remaining n points. As a simple example,
>>> three equidistant points on a 2-dimensional sphere (a circle), can be
>>> located 120 degrees apart. Any hints on how to do this in general for
>>> n-dimensions?

For n=1, the solution is two points at +/-1.

For n>1, take the solution for n-1 dimensions, centered at the origin. Add one dimension, and give all those points a value of 0 in the new dimension. Add a point with a value of 'x' for the new dimension, with all it's other coordinates set to 0. Calculate the distance of the new point from any of the old points, as a function of 'x'. Solve for the value of 'x' that puts the new point at the same distance from the old point, as the old point was from all of the other points. By symmetry, the new point will also be that same distance from all of the other old points. Find the center of the new set of points. Shift all the points by the amount needed to center them on the desired location. Shift all of the points outward from the center by the same factor, to get a hypersphere of the desired radius.

Example, for n=2:

Start with two points a <-1,0> and <+1,0>.
Add a third point at <0,x>. The distance from first point is
sqrt(1+x^2). The distance between first two points is 2, so x = sqrt(3).

The center of <-1,0>, <1,0> and <0,sqrt(3)> is at <0, 1/sqrt(3)>.

Shifting all the points to center at 0, we get <-1,-1/sqrt(3)>,
<1,-sqrt(3)/3> and <0,2/sqrt(3)>. The new circle has a radius of 2/sqrt(3).

Multiply by sqrt(3)/2, to get a circle of radius one, leaving us with

<-sqrt(3)/2, -1/2>, <sqrt(3)/2, -1/2>, <0,1>.

>> Unfortunately, when you go to dimension greater than two, there are
>> constraints on the number of 'equidistant' points you can have on a
>> sphere. For example, in 3-D, there are (only) five regular polyhedra,
>> so n can only have the values 4, 6, 8, 12, and 20 for a tetrahedron,
>> octahedron, cube, icosahedron, and dodecahedron.
>
>
> So is there any requirement that the tesselation produce a regular
> polyhedron?

Yes. If all of the points are to be equidistant from each other, then the object they trace out is necessarily a regular polyhedron. I fact, there is only one polyhedron in three dimensions where all of the vertices are equidistant from each other, and that's the tetrahedron. In general, the maximum number is 1 more than the number of dimension of the sphere.

All the edges of regular polyhedra are the same length, but that's not the same thing.

> Clearly it is possible to place *any* number of equidistant points on
> a sphere via an iterative approach.   As discussed on line, start
> with random placement of points, allow the points to repel each other,
> iterate until you reach the lowest energy configure.

That can't produce an equidistant set of points for any n more than 1 higher than the number of dimensions. In three dimensions it also can't produce a regular polyhedron for any n other than the ones that were listed above. Try it with 5 points on the surface of a three-dimensional sphere. The precise configuration you'll end up with depends upon the force law you use for the repulsion, but it won't be a regular polyhedron, and it certainly won't be equidistant.