
Subject: Multiplying very high with very low numbers: erfc * exp

Posted by [tho.siebert](#) on Thu, 03 Apr 2014 09:35:10 GMT

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Hello,

for my MCMC fitting program, I need to evaluate functions of the form (Gaussian with a one sided exponential tail towards lower x-values):

$$f(a,b,c,d) * \operatorname{erfc}(g(a,b,c,d)) * \exp(h(a,b,c,d)) := X * Y * Z = F$$

where f,g and h are certain functions of the parameters a,b,c and d.

It almost always happens that the numbers of these three factors are like:

$$F = X * Y * Z = 1e2 * 1e-999 * 1e1000 = 1e3$$

Which is a big problem since 1e-999 is represented as 0 and 1e1000 is represented as infinity, thus the result being 0, infinity or nan, but definitely not 1e3.

As a work-around, I went to log-space such that:

$$\begin{aligned} F &= \exp(\ln(F)) = \exp(\ln(X * Y * Z)) = \exp(\ln(X) + \ln(Y) + \ln(Z)) = \\ &= \exp(\ln(f(a,b,c,d)) + \ln(\operatorname{erfc}(g(a,b,c,d))) + \ln(\exp(h(a,b,c,d)))) := \\ &:= \exp(Q + W + E) \end{aligned}$$

Q and E are no problem to evaluate since f() is just a rational function and ln(exp(h())) is just h(). However, W = ln(erfc(g())) contains the same problem as above. If g() is far negative from 0, erfc(g()) is just 2 (and not e.g. 2 - 1e-99). If g() is far positive from 0, erfc(g()) is just 0, returning W as -Inf (as erfc(g()) should actually be something like 1e-99).

Now, I looked up several representations of the erfc() function in order to build something like a lnerfc - function. I have chosen the erfcc() function in Numerical recipes, Chapter 6, Special Functions (around page 214) which is also given in Wikipedia at http://en.wikipedia.org/wiki/Error_function#Numerical_approximation

This approximation has two major advantages:

- 1) It is represented as proportional to an exponential function, for which the ln can easily be calculated.
- 2) The fractional error is "everywhere less than 1.2e-7".

Including all these work-arounds, F = X * Y * Z can be calculated to a good enough precision (for me).

However (again), as you might already think of, it takes a while to calculate F. In a MCMC run, this function has to be evaluated over and over again. If there is more than one such a function present in my data (say N), I need to fit, i.e. evaluate something like:

$$\sum(F_i, i=0..N)$$

over and over again (typically N = 20..30).

To put it in a nutshell:

I am looking for a speed-up to calculate $W = \ln(\operatorname{erfc}(g(a,b,c,d)))$.

I know that I can calculate the erfc - function by:

$$\operatorname{erfc}(x) = 1 - \operatorname{sgn}(x) * \operatorname{igamma}(0.5, x^2)$$

where igamma is the incomplete gamma-function.

Unfortunately, there is no LNIGAMMA - function in IDL, as for the complete gamma-function (LNGAMMA). As this does not necessarily have to work good then because of the "1 -".

I hope you understand the problem and are not overwhelmed by this wall of text.

I appreciate any suggestions.

Cheers,
Thomas
