
Subject: Re: determining if a point is "inside" or "outside" a shape

Posted by [Andrew Kraus](#) on Mon, 18 Oct 1999 07:00:00 GMT

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A method I have used extensively in the past is less elegant, but works like a charm. Create a pixmap of the entire window. Paint it black (ERASE), or any intensity value for that matter, then POLYFILL your coordinates with a different color. You can then check the color of the point you are testing. Be sure to color the coordinates (boundary) as well so you know if the point lies on the boundary itself. Pixmap and polyfill are 2 of the faster functions in IDL, so this goes very quickly. You can see how this handles any shape, except donuts require a bit more thought.

Best of luck,

AK

dmmarshall@ivory.trentu.ca wrote in message ...

> I have an interesting problem that I'm betting someone has solved already
> in some form or another.

>

> I have a set of x,y coordinates that describe a polygon, 2d in my case.

> How do I tell if another x,y point is inside the boundary of my polygon?

>

> I had an idea to take the center of my polygon and extend a line from it to
> the point and beyond, seeing if I cross the perimeter an odd number of
> times.

>

> Any other suggestions?

>

> Dave.

>

> David Marshall Physics Dept. Trent University

> dmmarshall@remove.this.ivory.trentu.ca Peterborough Ontario Canada

>

Subject: Re: determining if a point is "inside" or "outside" a shape

Posted by [Med Bennett](#) on Mon, 18 Oct 1999 07:00:00 GMT

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I have solved this problem with the included two functions. It works by first creating a point outside the polygon, then creating a line from the outside point to the test point. Then, the algorithm counts the number of intersections between the test line and the lines making up the polygon. An odd number

indicates that the test point is inside the polygon, while an even number indicates a point outside the polygon. This method ensures a correct result even in the case of polygons with concave sides, etc. The function `inpoly` requires the `line_int` function, and returns an array with the same number of elements as the

number of data points, with a 1 for points inside the polygon and 0 for points outside the polygon. Hope that this helps and that you can decipher my lousy code. There is a problem if two adjacent points have the same X coord (slope of one line segment is infinite) - this needs to be fixed!

=====

```
function inpoly,polyxy,dataxy
```

```
; procedure to determine which points in dataxy
; fall inside a given polygon
; polygon is defined by series of vertices given
; in polyxy
```

```
polyxy = double(polyxy)
dataxy = double(dataxy)
```

```
;determine polygon min and max x and y values
```

```
pxmax = max(polyxy(0,*))
pxmin = min(polyxy(0,*))
pymax = max(polyxy(1,*))
pymin = min(polyxy(1,*))
```

```
;determine size of input arrays
```

```
dsize = size(dataxy)
if dsize(0) eq 1 then dpts = 1 else dpts = dsize(2)
datain = intarr(dpts)
```

```
psize = size(polyxy)
ppts = psize(2)
int = intarr(ppts)
```

```
; determine which points lie inside polygon bounding box
```

```
w = where((dataxy(0,*) lt pxmax) and (dataxy(0,*) gt pxmin) and $
  (dataxy(1,*) lt pymin) and (dataxy(1,*) gt pymin),c)
```

```
for i = 0,c-1 do begin ;loop through data
```

```
  testpt = [pxmin-(.1*(pxmax-pxmin)),pymax+(.1*(pymax-pymin))]
  l1 = [[testpt],[dataxy(*,w(i))]] ;line 1 is from testpt to data point
```

```
;loop through polygon edge segments - last point in polygon array must be same
as
first!
```

```
    for j = 0,ppts-2 do begin
        if ( polyxy(0,j) ne polyxy(0,j+1) or polyxy(1,j) ne polyxy(1,j+1) ) then
begin
            l2 = [[polyxy(*,j)],[polyxy(*,j+1)]] ;second line connects two vertices

            int(j) = line_int(l1,l2) ;calculate line intersection
        endif
    endfor
```

```
    nint = total(int) ;total no. of intersections
```

```
;even if point is outside polygon; odd if inside
    if (nint/2 eq fix(nint/2)) then datain(w(i)) = 0 $
        else datain(w(i)) = 1
```

```
endfor
print,total(datain),' points inside polygon.'
return,datain
```

```
end
```

```
=====
function line_int,l1,l2
```

```
; calculate line equations
```

```
; print,l1,l2
```

```
if ( l1(0,0) ne l1(0,1) ) then begin
    slope1 = (l1(1,1)-l1(1,0))/(l1(0,1)-l1(0,0))
    yint1 = l1(1,0) - slope1*l1(0,0)
; print,'m1',slope1,'b1',yint1
endif else begin
    if ((l1(0,0) lt max(l2(0,*))) and (l1(0,0) gt min(l2(0,*)))) then return,1 $
    else return,0
endelse
```

```
if ( l2(0,0) ne l2(0,1) ) then begin
    slope2 = (l2(1,1)-l2(1,0))/(l2(0,1)-l2(0,0))
    yint2 = l2(1,0) - slope2*l2(0,0)
; print,'m2',slope2,'b2',yint2
endif else begin
    if ((l2(0,0) lt max(l1(0,*))) and (l2(0,0) gt min(l1(0,*)))) then return,1 $
```

```

    else return,0
endelse

; if lines are parallel, no intersection

if (slope1 ne slope2) then begin

xintersect = (yint2-yint1)/(slope1-slope2)
;print,'xintersect',xintersect
endif else return,0

if ( (xintersect lt max(l1(0,*))) and (xintersect gt min(l1(0,*))) $
and (xintersect lt max(l2(0,*))) and (xintersect gt min(l2(0,*)))) then
return,1
$
    else return,0

end
=====

```

"Martin LUETHI GL A8.1 2-4092" wrote:

```

> Dear all
>
> Is there a simple and fast way to find the indices of array elements, which
> are inside of a boundary (in 2 dimensions). Let's say coord(npoints, 2) is an
>
> array of coordinates in the plane and bound(nbound, 2) is the array of a
> bounday polygon. The coordintes are not on a grid (otherwise one could use
> polyfillv (PV-Wave).
>
> Thank you for any suggestion!
>
> Martin
>
> --
> =====
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> Glaciology Section    Fax. +41 1 632 11 92
> VAW ETH Zuerich
> CH-8092 Zuerich       mail luthi@vaw.baum.ethz.ch
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> =====

```

dmarshall@ivory.trentu.ca wrote:

```

> I have an interesting problem that I'm betting someone has solved already

```

> in some form or another.
>
> I have a set of x,y coordinates that describe a polygon, 2d in my case.
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> I had an idea to take the center of my polygon and extend a line from it to
> the point and beyond, seeing if I cross the perimeter an odd number of
> times.
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> Any other suggestions?
>
> Dave.
>
> David Marshall Physics Dept. Trent University
> dmarshall@remove.this.ivory.trentu.ca Peterborough Ontario Canada

Subject: Re: determining if a point is "inside" or "outside" a shape
Posted by [Job von Rango](#) on Thu, 21 Oct 1999 07:00:00 GMT
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There is another analytic way to solve the problem:

How to determine whether a point lies inside an arbitrary 2-dimensional polygon or not?

IDEA:

Look at the sum of all angles tended between all pairs of neighbored vertices the polygon and the given point.

IN DETAIL:

Assume we have the n vertices of polygons ordered at the positions:

v_1, v_2, \dots, v_n

The given point is denoted by p.

Now connect the given point p with all n neighbored vertices, and get the n vectors beginning in p and ending in the vertices:

$vec_1 = \text{vector}(p, v_1)$

...

$vec_n = \text{vector}(p, v_n)$

Now add all n angles tended between the 2 vectors vec_i and $vec_{(i+1)}$:

$angle_i = \text{angle}(vec_i, vec_{(i+1)})$

and calculate the sum:

$$\text{anglesum} = \sum_{i=1}^n \text{angle}_i$$

(Use vec_1 again for $\text{vec}_{(n+1)}$ in the last angle_n)

RESULT:

$$\text{anglesum} = \begin{cases} 2\pi & \text{inside} \\ -\text{angle} & \text{if p is the polygon} \\ 0 & \text{outside} \end{cases}$$

IMPORTANT:

Take into account the correct sign of the angles
and use the vertices in ascending order!

Use e.g. the vector crossproduct:

$$\text{vec}_i \times \text{vec}_{(i+1)}$$

in order to retrieve the absolute value $|\dots|$ for the angle:

$$|\text{angle}_i| = \arcsin(|\text{vec}_i \times \text{vec}_{(i+1)}|)$$

The valid sign for angle_i is given by looking at the scalar product
of the crossproduct from above and a fixed vector vec_{perp} , perpendicular
to the area of the polygon:

$$\text{sign}_i = \frac{\text{vec}_{\text{perp}} \cdot (\text{vec}_i \times \text{vec}_{(i+1)})}{|\text{vec}_{\text{perp}} \cdot (\text{vec}_i \times \text{vec}_{(i+1)})|}$$

Now we get the correct value for all angles:

$$\text{angle}_i = \text{sign}_i * |\text{angle}_i|$$

REMARK:

Examine the case of a given point inside the polygon, but very near
to the edge between two vertices v_i and $v_{(i+1)}$. The contributing
angle will be:

$$\text{angle}_i = +\pi - \epsilon$$

(where epsilon denotes a small positive angle.)
If we shift p over the edge to the outside of the polygon
(but very near again to the edge)
the contributing angle will be:

$$\text{angle}_i = -\pi + \epsilon$$

The switch of the sign is the reason for the discontinuity ($2\pi \leftrightarrow 0$)
of anglesum for points moving from inside to the outside of the polygon!

----- --
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Subject: Re: determining if a point is "inside" or "outside" a shape
Posted by [Job von Rango](#) on Thu, 21 Oct 1999 07:00:00 GMT
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IDEA:

Look at the sum of all angles tended between all pairs of neighbored
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Now connect the given point p with all n neighbored
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in the vertices:

$\text{vec}_1 = \text{vector}(p, v_1)$

...

$\text{vec}_n = \text{vector}(p, v_n)$

Now add all n angles tended between the 2 vectors `vec_i` and `vec_(i+1)`:

```
angle_i = angle(vec_i,vec_(i+1))
```

and calculate the sum:

```

n
anglesum = sum angle_i
i=1

```

(Use `vec_1` again for `vec_(n+1)` in the last `angle_n`)

RESULT:

```

|_ 2*pi    inside
anglesum = -|    if p is    the polygon
|_ 0        outside

```

IMPORTANT:

Take into account the correct sign of the angles
and use the vertices in ascending order!
Use e.g. the vector crossproduct:

```
vec_i x vec_(i+1)
```

in order to retrieve the absolut value `|...|` for the angle:

```
|angle_i| = inv sin( |vec_i x vec_(i+1)| )
```

The valid sign for `angle_i` is given by looking at the scalar product of the crossproduct from above and a fixed vector `vec_perp`, perpendicular to the area of the polygon:

```

vec_perp * ( vec_i x vec_(i+1) )
sign_i = -----
| vec_perp * ( vec_i x vec_(i+1) ) |

```

Now we get the correct value for all angles:

```
angle_i = sign_i * |angle_i|
```

REMARK:

Examine the case of a given point inside the polygon, but very near to the edge between two vertices `v_i` and `v_(i+1)`. The contributing

angle will be:

$$\text{angle}_i = +\pi - \epsilon$$

(where epsilon denotes a small positive angle.)

If we shift p over the edge to the outside of the polygon

(but very near again to the edge)

the contributing angle will be:

$$\text{angle}_i = -\pi + \epsilon$$

The switch of the sign is the reason for the discontinuity ($2\pi \leftrightarrow 0$) of anglesum for points moving from inside to the outside of the polygon!

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Subject: Re: determining if a point is "inside" or "outside" a shape

Posted by [Randall Frank](#) on Fri, 22 Oct 1999 07:00:00 GMT

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I would suggest you read the Graphics FAQ on this issue and also check Graphics Gem (I think volume 1) for a more detailed explanation of this problem. The upshot is that there really are three core methods and many variants. In general, you can sum angles, sum signed areas or clip a line. There are good code examples of all these approaches on the net which can be coded into IDL very quickly. It also depends on how you intend to use the function. If, you are going to repeatedly test many points, you are better off using one of the sorted variants of the line clipping techniques. In general, the line clipping techniques are the fastest on the average, but have poor worst case performance without the sorting overhead. The angle sum is one of the slowest methods unless you can get creative and avoid the transcendentals (and you can). The area sum approach generally falls inbetween. In IDL code, I believe you can vectorize the latter with some setup overhead, making it the fastest for .pro code when testing multiple points with one point per call.

FWIW.

Job von Rango wrote:

```
> Ther is another analytic way to solve the problem:
>
> How to determine wether a point lies inside an arbitrary 2-dimensional
> polygon
> or not?
>
> IDEA:
> Look at the sum of all angles tended between all pairs of neighboured
> vertices
> the polygon and the given point.
>
> IN DETAIL:
> Assume we have the n vertices of polgons ordered at the positions:
>   v_1, v_2, ... ,v_n
> The given point is denoted by p.
>
> Now connect the given point p with all n neighboured
> vertices, and get the n vectors beginning in p and ending
> in the vertices:
>
>   vec_1 = vector(p_0, v_1)
>   ...
>   vec_n = vector(p_0, v_n)
>
> Now add all n angles tended between the 2 vectors vec_i and vec_(i+1):
>
>   angle_i = angle(vec_i,vec_(i+1))
>
> and calculate the sum:
>
>   n
>   anglesum = sum  angle_i
>   i=1
>
> (Use vec_1 again for vec_(n+1) in the last angle_n)
>
> RESULT:
>
>   | 2*pi   inside
>   anglesum = -|   if p is   the polygon
>   | 0       outside
>
> IMPORTANT:
> Take into account the correct sign of the angles
> and use the vertices in ascending order!
> Use e.g. the vector crossproduct:
>
```

```

>     vec_i x vec_(i+1)
>
> in order to retrieve the absolut value |...| for the angle:
>
>     |angle_i| = inv sin( |vec_i x vec_(i+1)| )
>
> The valid sign for angle_i is given by looking at the scalar product
> of the crossproduct from above and a fixed vector vec_perp, perpendicular
> to the area of the polygon:
>
>     vec_perp * ( vec_i x vec_(i+1) )
>     sign_i = -----
>               | vec_perp * ( vec_i x vec_(i+1) ) |
>
> Now we get the correct value for all angles:
>
>     angle_i = sign_i * |angle_i|
>
> REMARK:
> Examine the case of a given point inside the polygon, but very near
> to the edge between two vertices v_i and v_(i+1). The contributing
> angle will be:
>
>     angle_i = + pi - epsilon
>
> (where epsilon denotes a small positive angle.)
> If we shift p over the edge to the outside of the polygon
> (but very near again to the edge)
> the contributing angle will be:
>
>     angle_i = - pi + epsilon'
>
> The switch of the sign is the reason for the discontinuity (2*pi <-> 0)
> of anglesum for points moving from inside to the outside of the polygon!
>
> -----
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>
--
rjf.

```

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