Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by davidf on Tue, 22 Feb 2000 08:00:00 GMT
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Craig Markwardt (craigmnet@cow.physics.wisc.edu) writes:

> P.S. A little research is all it takes!

It also helps if you are under-employed enough
to have the time. :-)

But I do appreciate you folks running this
down for us.

Cheers,

David

P.S. But that theorem though...it's almost enough
to make you want to go back to school and study math,
isn't it. :-)

--
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Coyote's Guide to IDL Programming: http://www.dfanning.com/
Toll-Free IDL Book Orders: 1-888-461-0155

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Craig Markwardt on Tue, 22 Feb 2000 08:00:00 GMT
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Med Bennett <mbennett@indra.com> writes:

> This is an interesting problem and I was hoping that someone would have
> provided a slick answer by now. I started searching the Web and came up
> with the following. It seems as though you could triangulate your points
> and then use the theorem presented below:
> [... deleted ...]

This was discussed a little bit in August 1999 (see "area enclosed by
a polygon on a sphere" on www.deja.com). The tricky part of course is
computing the correct angles. Struan Gray wondered if there was a
utility routine in the idlastro library which could help.

Try GCIRC of idlastro, but also these two from the "JHU/APL/S1R usr

Library".

SPHGC Find intersections of two great circles on sphere.
SPHIC Compute intersection points of two circles on a unit sphere.

I found these here:
http://www.astro.washington.edu/deutsch/idl/

Craig

P.S. A little research is all it takes!

--
--- --------------
Craig B. Markwardt, Ph.D.   EMAIL: craigmnet@cow.physics.wisc.edu
Astrophysics, IDL, Finance, Derivatives | Remove "net" for better response
--- --------------

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Med Bennett on Tue, 22 Feb 2000 08:00:00 GMT
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This is an interesting problem and I was hoping that someone would have provided a slick answer by now. I started searching the Web and came up with the following. It seems as though you could triangulate your points and then use the theorem presented below:

http://www.geom.umn.edu/docs/doyle/mpls/handouts/node16.html

Geometry on the sphere

We want to explore some aspects of geometry on the surface of the sphere. This is an interesting subject in itself, and it will come in handy later on when we discuss Descartes's angle-defect formula.

Discussion

Great circles on the sphere are the analogs of straight lines in the plane. Such curves are often called geodesics. A spherical triangle is a region of the sphere bounded by three arcs of geodesics.

1.Do any two distinct points on the sphere determine a unique geodesic? Do two distinct geodesics intersect in at most one point?
2.Do any three `non-collinear' points on the sphere determine a unique triangle? Does the sum of the angles of a spherical triangle always equal pi? Well, no. What values can the sum of the angles take on?

The area of a spherical triangle is the amount by which the sum of its angles exceeds the sum of the angles (pi) of a Euclidean triangle. In fact, for any spherical polygon, the sum of its angles minus the sum of the angles of a Euclidean polygon with the same number of sides is equal to its area.

A proof of the area formula can be found in Chapter 9 of Weeks, The Shape of Space.

Kyong-Hwan Seo wrote:

> I am looking for a way to calculate area on sphere.
> I have arrays of the position of the connected points (i.e, longitudes
> and latitudes).
> If somebody has an idl program, please let me know.
> Thanks in advance,
>
> K.H. Seo

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Ben Tupper on Wed, 23 Feb 2000 08:00:00 GMT

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Kyong-Hwan Seo wrote:

> I am looking for a way to calculate area on sphere.
> I have arrays of the position of the connected points (i.e, longitudes
> and latitudes).
>

Hello,

The following may be helpful if you have only three verticies enclosing the area.

This is from Bronshtein and Semendyayev, A GUIDE BOOK TO MATHEMATICS, Springer-Verlag, 1973.

"A fundamental property of a spherical triangle is that the sum of its angles A+B+C is always greater than 180 degrees. The difference, (A+B+C) - pi= delta , expressed in radians is called the spherical excess of the given spherical triangle. The area of a spherical triangle is S=R^2 * delta, where R is the radius of the sphere."

Ben

--
Ben Tupper

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tupper@seadas.bigelow.org

pemaquidriver@tidewater.net

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Tim Cross on Thu, 24 Feb 2000 08:00:00 GMT
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Med Bennett wrote:
>
> Great circles on the sphere are the analogs of straight lines in the
> plane. Such curves are often called geodesics. A spherical triangle is a
> region of the sphere bounded by three arcs of geodesics.
>
> 1.Do any two distinct points on the sphere determine a unique geodesic?

Yes. Years ago, I could prove it.

> Do two distinct geodesics intersect in at most one point?

Fuzzy language, but they intersect at zero points, one point,
or along some geodesic that is a subset of both. Years ago, ...

> 2.Do any three `non-collinear' points on the sphere determine a unique
> triangle?

Two unique triangles - the obvious one that covers < half the sphere,
and the slightly less obvious one that covers the rest of the sphere.
Two unique triangles - it that English?

> Does the sum of the angles of a spherical triangle always equal
> pi? Well, no. What values can the sum of the angles take on?

The small degenerate spherical triangle is a single point, and as
the area of the triangle approaches zero, the sum of the angles
approaches pi, i.e., things get more planar, and more like, say,
a football field cut diagonally, and less like, say, the state
of Colorado cut diagonally. The large degenerate spherical triangle
is everything but the point, and as the area of the triangle
approaches 4pi*r^2 (the area of the sphere), the three angles
approach 2pi, for a total of 6pi.

Do I have a formula for calculating the area of a spherical

triangle? Not offhand. And I've got a job I should probably
get back to... :-)

--
Tim Cross timc@boulder.vni.com 303-245-5393
Visual Numerics, Inc.
5775 Flatiron Parkway, Suite 220
Boulder CO 80301 USA
http://www.vni.com
My opinions, etc.

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Tim Cross on Mon, 28 Feb 2000 08:00:00 GMT

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Stein Vidar Hagfors Haugan wrote:
>
> Tim Cross <timc@boulder.vni.com> writes:
>
>> Med Bennett wrote:
>>>
>>> Great circles on the sphere are the analogs of straight lines in the
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>>> region of the sphere bounded by three arcs of geodesics.
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>>> 1.Do any two distinct points on the sphere determine a unique geodesic?
>>
>> Yes. Years ago, I could prove it.
>
> Funny, I'd like to see that proof.
>
> What's the unique geodesic connecting the north and south poles :-?

OUCH!!!! See how fast the memory fades?

--
Tim Cross timc@boulder.vni.com 303-245-5393
Visual Numerics, Inc.
5775 Flatiron Parkway, Suite 220
Boulder CO 80301 USA
http://www.vni.com
My opinions, etc.

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by Stein Vidar Hagfors H[1] on Mon, 28 Feb 2000 08:00:00 GMT

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Tim Cross <timc@boulder.vni.com> writes:

> Med Bennett wrote:
>>
>> Great circles on the sphere are the analogs of straight lines in the
>> plane. Such curves are often called geodesics. A spherical triangle is a
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Funny, I'd like to see that proof.

What's the unique geodesic connecting the north and south poles :-?
--
Stein Vidar Hagfors Haugan
ESA SOHO SOC/European Space Agency Science Operations Coordinator for SOHO
NASA Goddard Space Flight Center, Email: shaugan@esa.nascom.nasa.gov

---

Subject: Re: CALCULATION OF AREA ON A SPHERE
Posted by James Kuyper on Thu, 23 Mar 2000 08:00:00 GMT

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Tim Cross wrote:
>
> Med Bennett wrote:
>>
>> Great circles on the sphere are the analogs of straight lines in the
>> plane. Such curves are often called geodesics. A spherical triangle is a
>> region of the sphere bounded by three arcs of geodesics.
>>
>> 1.Do any two distinct points on the sphere determine a unique geodesic?
>
> Yes. Years ago, I could prove it.

Not true for points on opposite points of the sphere. If you want to
make a close connection between spherical geometry and planar geometry,
you have to replace a "line" with a "great circle arc", and a "point"
with a "pair of diametrical opposite points". With those substitutions,
spherical geometry becomes formally identical to planar geometry, except
for the parallel postulate.

>> Do two distinct geodesics intersect in at most one point?
>

> Fuzzy language, but they intersect at zero points, one point,
> or along some geodesic that is a subset of both. Years ago, ...

Geodesics of length equal to 1/2 the circumference of the sphere can intersect at two points, if those are their starting and ending points.

...
> Two unique triangles - it that English?

Yes.

...
> Do I have a formula for calculating the area of a spherical
> triangle? Not offhand. And I've got a job I should probably
> get back to... :-)

IIRC, the sum of the angles is linearly related to the area enclosed. I'll leave derivation of slope and intercept as an excersize for the reader. Hint: consider the interior and exterior area enclosed by a spherical triangle whose sides are vanishingly small, so that the surface seems perfectly flat in it's vicinity. That allows you to use ordinary plane geometry to calculate the sum of the angles.