
Subject: Re: Meaning of outer product

Posted by [James Kuyper](#) on Sat, 13 Jul 2002 18:17:05 GMT

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Paul Sorenson wrote:

>
> Greetings,
>
> IDL documentation says: "Note - If A and B arguments are vectors, then C =
> MATRIX_MULTIPLY(A, B) is a matrix with $C_{ij} = A_i B_j$. Mathematically, this
> is equivalent to the outer product. . . ." But I'm having difficulty
> reconciling this with my understanding of outer product. . .
>
> $c.x = a.y*b.z - a.z*b.y$
> $c.y = a.z*b.x - a.x*b.z$
> $c.z = a.x*b.y - a.y*b.x$
>
> ... which yields a vector (c) instead of a 2D array. Can anyone shed some
> light on this?

The outer product you are referring to is also called the cross-product, and is different from the outer product that the IDL documentation is talking about. However, the two concepts are related. In general, the outer product of two vectors could be written as:

$$OP = \begin{bmatrix} a.x*b.x & a.x*b.y & a.x*b.z \\ a.y*b.x & a.y*b.y & a.y*b.z \\ a.z*b.x & a.z*b.y & a.z*b.z \end{bmatrix}$$

You can create such an outer product by matrix multiplication of column vector by a row vector (matrix multiplication of a row vector by a column vector gives their dot product).

While 'OP' has 9 different elements, they were calculated from only six different original numbers. The entire information content of that array can be summarized by forming two other matrices:

$$SYM = (OP + \text{transpose}(OP))/2 \text{ Symmetrized}$$
$$ASYM = (OP - \text{transpose}(OP)) \text{ Anti-symmetrized}$$

The ASYM matrix necessarily has 0 on it's diagonal elements, and the off-diagonal elements come in pairs that are exactly equal in size, and opposite in sign. Therefore, it can be described by exactly three independent numbers. The cross-product is a special rearrangement of those three numbers to create a pseudo-vector. It's a pseudo-vector because it can be rotated the same way as a real vector, but remains unchanged by mirror reflections. Mathematically, it's called the "dual" of the anti-symmetrized outer product.

I'm not sure how the cross-product ended up also being called the "outer product", but it's probably because of this connection. For some purposes, the three numbers in the cross-product represent all of the useful information that's contained in the outer product.

Subject: Re: Meaning of outer product

Posted by [MKatz843](#) on Sat, 13 Jul 2002 21:30:20 GMT

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That's the "cross-product" $c = a \times b$ you've written (above).
As you know, the "inner product" of two 3-element vectors is a scalar,
also known as the "dot-product"

$c = a \cdot b = a.x*b.x + a.y*b.y + a.z*b.z$

The inner product is written as a row vector times a column vector.

The "outer product" of two three-element vectors is a 3x3 matrix

C =

($a.x*b.x$ $a.x*b.y$ $a.x*b.z$)

($a.y*b.x$ $a.y*b.y$ $a.y*b.z$)

($a.z*b.x$ $a.z*b.y$ $a.z*b.z$) (I hope this isn't the transpose!)

it's usually written as a column vector times a row vector.

Remember that vector and matrix multiplications aren't necessarily
commutative: $a \text{ times } b \neq b \text{ times } a$, necessarily.

M. Katz

Subject: Re: Meaning of outer product

Posted by [Paul Sorenson](#) on Fri, 19 Jul 2002 01:29:05 GMT

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Thanks, James and M. Katz. Your responses really helped clear up this issue

for me. To summarize: the IDL documentation is correct in referring to $C_{ij} = A_i B_j$ as the outer product. Other resources that refer to the cross product as the "outer product" are using the term more loosely.

James...

```
> OP = [[a.x*b.x, a.x*b.y, a.x*b.z],  
> [a.y*b.x, a.y*b.y, a.y*b.z],  
> [a.z*b.x, a.z*b.y, a.z*b.z]]
```

... I think you meant

```
OP = [[a.x*b.x, a.x*b.y, a.x*b.z],  
[a.y*b.x, a.y*b.y, a.y*b.z],  
[a.z*b.x, a.z*b.y, a.z*b.z]]
```

-Paul Sorenson

"M. Katz" <MKatz843@onebox.com> wrote in message
news:4a097d6a.0207131330.1a6c1835@posting.google.com...

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> The "outer product" of two three-element vectors is a 3x3 matrix

> C =

> $(a.x*b.x \ a.x*b.y \ a.x*b.z)$

> $(a.y*b.x \ a.y*b.y \ a.y*b.z)$

> $(a.z*b.x \ a.z*b.y \ a.z*b.z)$ (I hope this isn't the transpose!)

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- > M. Katz

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