Subject: Re: Volume by four vectors
Posted by ed on Wed, 13 Oct 2004 21:02:33 GMT
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Dear Sander,

Interesting geometry problem. The volume you're talking about is an irregular polyhedron--the generalization of a cube. You can dissect the volume into
irregular tetrahedra, each bounded by 4 triangles, two triangles on each face.
Pick an origin inside of the hexahedron, and since there are 6 faces, there
will be 12 (irregular) tetrahedra, each with a common vertex at the origin.
Obviously the location of the origin is irrelevant, but if it is outside of
the hexahedron, some of the tetrahedra will have negative volumes.

The volume of a general tetrahedron was calculated by Piero della Francesca
in the 1400s. You can read about this formula at
http://www.mathpages.com/home/kmath424.htm
but it boils down to this (quoted from the above link)

...a 3-dimensional analogue
of Heron's formula for the volume of a general tetrahedron with edges a,b,c,d,e,f, taken in opposite pairs (a,f), (b,e), (c,d). Letting A,B,..,F denote the *squares* of these respective edge lengths, his formula was

144 V^2 = - ABC - ADE - BDF - CEF + ACD + BCD + ABE + BCE
+ BDE + CDE + ABF + ACF + ADF + CDF + AEF + BEF
- CCD - CDD - BBE - BEE - AAF -

So from your vertex vectors, calculate A,B,C,D,E & F for each tetrahedron, calculate V for each of the 12 tetrahedrons, and add them up. It's as "simple" as that.

Anyone for coding this in IDL?

Ed Schmahl
University of Maryland

Sander Roosendaal <sander@wereldraadsel.nl> wrote in message news:<5rgk32-p13.ln1@wereldraadsel.demon.nl>...
> Dear readers,
>

> I have the following question, which I have to solve in PV-Wave.
>
> Given four 3D vectors a=[a1,a2,a3] b=[b1,b2,b3] c=[c1,c2,c3] and
> d=[d1,d2,d3], I want to calculate the volume defined by
>
> i*a + j*b + k*c + l*d with i,j,k,l between 0 and 1.
>
> There must be a solution somewhere, or a hint to a solution. I just couldn't
> find it, and I am too lazy to derive it :-)
>
> Thank you.

Subject: Re: Volume by four vectors
Posted by Sander Roosendaal on Thu, 14 Oct 2004 16:52:34 GMT
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Many thanks, Ed.
Just the hint I needed.

Ed Schmahl wrote:

> Dear Sander,
>
> Interesting geometry problem. The volume you're talking about is an
> irregular polyhedron--the generalization of a cube. You can dissect
> the volume into
> irregular tetrahedra, each bounded by 4 triangles, two triangles on
> each face.
> Pick an origin inside of the hexahedron, and since there are 6 faces,
> there
> will be 12 (irregular) tetrahedra, each with a common vertex at the
> origin.
> Obviously the location of the origin is irrelevant, but if it is
> outside of
> the hexahedron, some of the tetrahedra will have negative volumes.
>

--
hartelijke groeten,

Sander
sander@wereldraadsel.nl

Subject: Re: Volume by four vectors
Posted by Sander Roosendaal on Thu, 14 Oct 2004 19:11:08 GMT
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Dear Ed,

I appreciate your help.

For my problem:
"Given four 3D vectors a=[a1,a2,a3] b=[b1,b2,b3] c=[c1,c2,c3] and
d=[d1,d2,d3], I want to calculate the volume defined by
i*a + j*b + k*c + l*d with i,j,k,l between 0 and 1."

You responded:

> Interesting geometry problem. The volume you're talking about is an
> irregular polyhedron--the generalization of a cube. You can dissect
> the volume into
> irregular tetrahedra, each bounded by 4 triangles, two triangles on
> each face.
> Pick an origin inside of the hexahedron, and since there are 6 faces,

There are more than 6 faces.

I calculate a 16 points. Only some of them define the polyhedron.

O(rigin),a,b,c,d,a+b,a+c,a+d,b+c,b+d,c+d, a+b+c,a+b+d,a+c+d,b+c+d, a+b+c+d

Example:
If a=[1,0,0]; b=[0,1,0]; c=[0,0,1], d=[0.01,0.01,0]
The resulting volume will be very close to 1. Actually, the 16 points are
*1: (0,0,0)
*2: (1,0,0)
*3: (0,1,0)
*4: (0,0,1)
5: (0.01,0.01,0)
6: (1,1,0)
*7: (1,0,1)
*8: (1.01,0.01,0)
*9: (0,1,1)
*10: (0.01,1.01,0)
11: (0.01,0.01,1)
12: (1,1,1)
*13: (1.01,1.01,0)
*14: (1.01,0.01,1)
*15: (0.01,1.01,1)
*16: (1.01,1.01,1)

The ones marked with * are on the outside (12 in total). This polyhedron has

8 faces.

--
hartelijke groeten,

Sander
sander@wereldraadsel.nl

If this helped, please take the time to rate the value of this post
http://rate.affero.net/sander/

Subject: Re: Volume by four vectors
Posted by ed on Sun, 17 Oct 2004 14:35:45 GMT
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Sander,

Uhoh! It's a volume with 5 vertices, obviously not cube-like in any way. But I guess you can dissect it into irregular tetrahedrons and use that Francesca formula for each one. The volume of the original solid may not be convex in general, which might lead to negative tetrahedral volumes, but I suspect, even then the sum of the volumes will be correct.

Ed

Sander Roosendaal <sander@wereldraadsel.nl> wrote in message news:<kfn242-pe3.ln1@wereldraadsel.demon.nl>...
> Many thanks, Ed.
> Just the hint I needed.
>
> Ed Schmahl wrote:
>
>> Dear Sander,
>>
>> Interesting geometry problem. The volume you're talking about is an
>> irregular polyhedron--the generalization of a cube. You can dissect
>> the volume into
>> irregular tetrahedra, each bounded by 4 triangles, two triangles on
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>>