Subject: Re: Fourier Transform when intervals are not uniform
Posted by Timm Weitkamp on Tue, 11 Oct 2005 08:48:34 GMT
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Sid,

Your problem is the same encountered in X-ray computerized tomography. The most common technique used to solve it is known as "filtered backprojection" (FBP). In tomography, it is the Fourier-space data that are sampled on a radial grid and need to be transformed back into real space. The fundamental idea behind FBP is to perform the interpolation from the radial grid to a Cartesian grid in real space rather than in Fourier space. This avoids artifacts, albeit at the expense of more computing time.

FBP works as follows: take your 2D radially-sampled data and

1) for each line of points that intersects the origin (i.e., each set of points with equal theta or theta+pi),

  a) apply a weight factor to each point that is essentially
    the distance of the point from the origin. (This is done to compensate for
    the non-uniform sampling density, which is highest near the origin.).
    This is the "filtering" part.

  b) Then, take the 1D Fourier transform of the line.

  c) Then, smear the result out in two dimensions.
    This is the "backprojection" part.

2) Finally, add up the smeared 2D images of all lines, obtained in step 1c, and you will get the 2D FT.

The problem is also known as an inverse Radon transform. IDL provides a routine named RADON, which carries out steps 1c and 2 for you in a more convenient and faster way than you would be able to do if you programmed these steps in IDL yourself.

Also, be aware that the above is only a gross description of FBP. Implementing the steps above will work, but there are details to know and respect in order to avoid artifacts, which I don't go into. There is a fabulous book on tomography that explains all of this in an extremely practical way: "Principles of Computerized Tomographic Imaging" by A. Kak and M. Slaney. You can download the whole book from www.slaney.org/pct/. The part that deals with your problem is in Chapter 3, pages 56-74.

Good luck
Timm

Timm Weitkamp
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Sid wrote:
> Hello
>
> I am trying to (2-d) fourier transform when the data is sampled at
> non-uniform values of x and y (it is uniform in r and theta). I don't
> know if I should try to :
>
> 1. Write my own brute-force FT algorithm (I wrote one, using
> int_tabulate, gives me "sidelobes" in the 1-d FT of a gaussian)
>
> 2. Try and look at the algorithm of FFT in IDL to see if I can just
> take the FFT in radial co-ordinates.
>
> I looked up the Cooley-Tukey algorithm in Numerical Recipes, but
> doesn't seem like IDL uses that, since it works for any number of
> points.
>
> Does anyone know what algorithm the IDL FFT uses / how to get around
> this?
