Subject: Re: plot dirac delta function?
Posted by Paolo Grigis on Wed, 19 Jul 2006 11:52:34 GMT
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here's your discrete delta function:

x=fltarr(100)
x[50]=0.01
plot,x,psym=10

if you want a step function, do instead
x[50:99]=1

Ciao,
Paolo

Nic wrote:
> Hi all!
>
> I am new to IDL and learning how to do non analytic plots such as a
> dirac delta function or a finite square well. Does anybody have ideas
> of what tools I should use or keywords to get started in plotting
> these?
>
> thank you
>

Subject: Re: plot dirac delta function?
Posted by James Kuyper on Wed, 19 Jul 2006 15:31:02 GMT
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Nic wrote:
> Hi all!
>
> I am new to IDL and learning how to do non analytic plots such as a
> dirac delta function or a finite square well. Does anybody have ideas
> of what tools I should use or keywords to get started in plotting
> these?
>
> thank you

The Dirac delta function is defined by the following equations:

delta(x) = 0 if x ne 0

integral of delta(x) dx from x0 to x1 = 1, if x0 < 0 and x1 > 0

Notice that this definition fails to identify explicitly the value of delta(0). That's because there's no meaningful value that can be assigned to delta(0). The best you can do is to call it infinity, but even that's not quite right, for reasons that I don't remember right now.
Conceptually, you can consider delta(x) to be a member of a family of functions delta(x,e), with the following properties:

delta_e(x,e) =0 for x<=-e or x>=e
delta_e(0,e) = 1/e
delta_e(x,e) is linear for -e<=x<=0, and for 0<=x<=e

With this definition, you can think of delta(x) as the limit, as e->0, of delta_e(x,e); except that this limit is not well-defined.

What you need to do to make an expression containing a delta function meaningfull is to integrate it. For instance:

integral f(x)*delta(a*(x-x1)) dx from x0 to x2 =
        f(x1)/a if   x0<x1<x2
          0   if x1<x0<x2 or x0<x2<x1

Therfore, it's rather meaningless to try to plot the Dirac delta function itself. If, however, you insist on doing so, you need to scale the y axis so that it maps an infinite range of values into a finite range on your screen. One simple transformation with this property is y=logit(norm), where norm is a value that runs from 0 to 1, and logit(norm) = alog(norm/(1-norm)). You can implement this in IDL by defining a function to be used as a YTICKFORMAT option of a PLOT command:

```
FUNCTION logit_format, axis, index, value
   IF(value LE 0) THEN RETURN, '-INFINITY';
   IF(value GE 1.0) THEN RETURN, '+INFINITY'
   RETURN, STRING(alog(value/(1-value)), FORMAT='(G9.3)')
END
```

The Dirac delta function is almost never used without shifting it's center. Let x0 be the shifted center, and let xmin and xmax be the domain over which you wish to plot delta(x-x0). Then the following commands will plot it "sort of" correctly:

```
PLOT, [xmin, x0, x0, x0, xmax], [0.5, 0.5, 1.0, 0.5, 0.5],
YTICKFORMAT='logit_format'
```

If you want to plot another function f(x) with the same scaling, you must rescale the y values, as follows:

OPLOT, x, exp(f(x))/(1+exp(f(x)))

The Heaviside step function is much simpler. It's defined as:

H(x) = 0 for x<0
H(0) = 0.5
H(x) = 1.0 for x>0

It's almost never used without scaling and offsets, so I'll show how to
plot y1*H(x-x0)+y0:

PLOT, [xmin, x0, x0, x0, xmax], [y0, y0, 0.5*(y0+y1), y1, y1]

Subject: Re: plot dirac delta function?
Posted by Benjamin Hornberger on Wed, 19 Jul 2006 16:28:56 GMT
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kuyper@wizard.net wrote:
>
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>
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> delta(0). That's because there's no meaningful value that can be
> assigned to delta(0). The best you can do is to call it infinity, but
> even that's not quite right, for reasons that I don't remember right
> now.

I think that (the last paragraph) applies to the continuous case. In the
discrete case, it follows quite clearly from the two equations above
that the delta function must be an array which is 1 for the center (more
on that below) and zero otherwise. Two examples:

1. Convolution of a function (array) with a delta function must
reproduce the function (or shift if the delta function is not centered).
This is clearly fulfilled by an array which has a 1 in one element and
zeroes otherwise.

2. The Fourier Transform of a (centered) delta function is a constant.
The value of the constant depends somewhat on the definition of the FT
(in IDL, the forward FFT of a delta function as defined above is a
constant 1/N, N being the number of array elements, and the reverse FFT
is a constant 1).

You have to keep in mind that for frequency analysis, IDL assumes the zero frequency ("center") to be at the zero element in real AND Fourier space. I, like many others, prefer to shift all arrays by minus/plus N/2 before and after an FT and work on "centered" arrays, which makes the plots look more intuitive.

Subject: Re: plot dirac delta function?
Posted by James Kuyper on Wed, 19 Jul 2006 19:20:16 GMT
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Benjamin Hornberger wrote:
> kuyper@wizard.net wrote:
>>
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> that the delta function must be an array which is 1 for the center (more
> on that below) and zero otherwise. Two examples:

In my experience, the term "Dirac delta function" is restricted to what you call the continuous case. The corresponding thing for the discrete case is referred to as the Kroenecker delta. See, for example, < http://www.physics.umd.edu/courses/Phys374/fall04/hw/MP27.ht m>.

I think it's even less meaningful (though much easier!) to plot the kroenecker delta than the Dirac delta function. It's only defined on a discrete set of points, so drawing lines between the consecutive points of the plot would be inappropriate.

Subject: Re: plot dirac delta function?
Posted by swingnut on Mon, 24 Jul 2006 00:54:39 GMT
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FYI, while the definition can be approached a number of equivalent ways, the value of the Dirac IS "well-defined" at delta(x). Technically, it's "well-defined" at the value such that its argument is zero (which here is x=0).

The value is indeed infinity. At least, that's how it's used in physics.
