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Subject: Least square fitting

Posted by [MichaelT](#) on Tue, 20 May 2008 09:41:32 GMT

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Hi all,

I have a problem with a rather complicated function depending on four parameters which I try to find using least square fitting and I don't know exactly how to do it.

The basic problem is the following:

I have an astronomical image of a star field and try to relate the sky coordinates (right ascension, declination) of the stars to the pixel coordinates (x, y).

The function to relate the two depends on the not accurately known parameters focal length of the lens (f), the rotation of the field of view with respect of the north direction (beta) and the center sky coordinates of the image (bc, lc).

I know the sky coordinates of certain stars accurately (bs, ls) as well as their pixel coordinates (xs, ys) and the center pixel coordinates of the image (xc, yc).

Generally, the function is as follows (equivalent for ys', but the functions f1-f5 are slightly different):

$$xs' = f1(xc, f2(f, f3(ls, lc, bs, bc)), f4(ls, lc, bs, bc, f5(ls, lc, bs, bc)), beta)$$

The functions f1-f5 look rather complicated and contain many sines, cosines and acos etc.

I then did the following as I didn't know better:

First I minimized all this with respect of beta, then lc and bc and after that f starting with some initial values for the four parameters. Then I started all over again, beta, lc, bc, f (in a loop) until the difference between the known coordinates (xs, ys) and the calculated ones (xs', ys') reached a minimum.

However, the minimum deviation reached at the end of the loop pretty much depends on the initially chosen values of beta, lc, bc and f.

There certainly is a more optimal way to do the least square fitting than the one that I have chosen (partial derivatives etc.), but that f1 depends on f2 and f4 and these again depend on f3 and f5 is giving me some unsolvable problems.

How should I proceed? What would be a good method to minimize this?

Thanks, Michael

The main problem now is for me, that f1 depends on f2 and f4 and that f2 and f4 themselves depend on f3 and f5.

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Subject: Re: Least square fitting  
Posted by [wlandsman](#) on Tue, 20 May 2008 17:28:32 GMT  
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On May 20, 5:41 am, MichaelT <michael.theus...@googlemail.com> wrote:

> The basic problem is the following:  
> I have an astronomical image of a star field and try to relate the sky  
> coordinates (right ascension, declination) of the stars to the pixel  
> coordinates (x, y).

>  
I have performed a lot of this type of astrometry, and your problem seems very strange to me.

> The function to relate the two depends on the not accurately known  
> parameters focal length of the lens (f), the rotation of the field of  
> view with respect of the north direction (beta) and the center sky  
> coordinates of the image (bc, lc).

A minor note is that if you really have such a large distortions then a single beta does not make sense -- the rotation from North will vary slightly across the image.

>  
> Generally, the function is as follows (equivalent for ys', but the  
> functions f1-f5 are slightly different):

>  
>  $xs' = f1(xc, f2(f, f3(l_s, l_c, b_s, b_c)), f4(l_s, l_c, b_s, b_c, f5(l_s, l_c, b_s, b_c)), \beta)$

>  
> The functions f1-f5 look rather complicated and contain many sines,  
> cosines and acos etc.

>  
> I then did the following as I didn't know better:  
> First I minimized all this with respect of beta, then lc and bc and  
> after that f starting with some initial values for the four  
> parameters. Then I started all over again, beta, lc, bc, f (in a loop)  
> until the difference between the known coordinates (xs, ys) and the  
> calculated ones (xs', ys') reached a minimum.

>

I have no idea what your functions f1, f2, f3, f4, f5 might be, but do you really care about them? All you want is a function that maps the known coordinates to the calculated ones. Often what one does is to find a quadratic or cubic function that will linearize the x,y coordinates (i.e. so that they line up with RA, Dec)

$$x_p = x + axy + by^2 + c*x^2 + ..$$
$$y_p = y + dxy + ey^2 + f*x^2$$

You then use least squares to determine the a,b,c.. coefficients. This is the 'SIP' convention discussed in <http://ssc.spitzer.caltech.edu/postbcd/doc/shupeADASS.pdf> and used e.g. by the astrometry.net software for handling distortions. Of course, it would be nice to know the true functional form of the distortions but whatever functions you are using don't seem very useful. You can think of the quadratic or cubic equations as a Taylor series approximation to whatever the true functional form is.

--Wayne

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Subject: Re: Least square fitting  
Posted by [MichaelT](#) on Tue, 20 May 2008 18:44:40 GMT  
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> I have performed a lot of this type of astrometry, and your problem  
> seems very strange to me.

The way I proceed is described here:

[http://home.arcor-online.de/axel.mellinger/mwpan\\_web/mwpan\\_web.html](http://home.arcor-online.de/axel.mellinger/mwpan_web/mwpan_web.html)

It uses the true functional form of the conversion (ra, dec) -> (x,y).

So I am not so sure if we are both talking about the same thing here.

I am not after the distortions, yet (These seem to be small. The calculated x',y' positions deviate by only 0.5 pixels, on average, from the real ones - when the algorithm converges).

> you really care about them? All you want is a function that maps  
> the known coordinates to the calculated ones. Often what one does

What I want is a function that converts (ra, dec) into pixel coordinates (x, y). So I am not so sure if that is what you describe in the following. What you describe looks more like (x', y') -> (x, y) to me.

> is to find a quadratic or cubic function that will linearize the x,y  
> coordinates (i.e. so that they line up with RA, Dec)

>  
>  $x_p = x + ax + by^2 + c*x^2 + ..$   
>  $y_p = y + dxy + ey^2 + f*x^2$   
>  
> You then use least squares to determine the a,b,c.. coefficients.  
> This is the 'SIP' convention discussed  
in <http://ssc.spitzer.caltech.edu/postbcd/doc/shupeADASS.pdf> and used  
> e.g. by the astrometry.net software for handling distortions.

The webpage I posted above also discusses the elimination of the distortions and uses a cubic function of the form given by you above and as discussed in the paper.

Can I also use the cubic/quadratic function to map (ra, dec) to (x, y) or vice versa?

This is my first attempt as an amateur astronomer to deal with these things :-)  
Many thanks for your help.

Michael

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Subject: Re: Least square fitting  
Posted by [Marshall Perrin](#) on Tue, 20 May 2008 21:48:34 GMT  
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MichaelT <michael.theusner@googlemail.com> wrote:

> Hi all,  
>  
> I have a problem with a rather complicated function depending on four  
> parameters which I try to find using least square fitting and I don't  
> know exactly how to do it.  
>  
> The basic problem is the following:  
> I have an astronomical image of a star field and try to relate the sky  
> coordinates (right ascension, declination) of the stars to the pixel  
> coordinates (x, y).

So if you truly wish to know how to properly solve this particular mathematical problem, I will leave that to other wiser experts to answer.

If, however, what you really want is just to know the astrometric solution for your data, you may be interested to know about Astrometry.net, a research project by David Hogg at NYU and collaborators which intends to solve this particular problem once and for all, for everyone. You feed their web service arbitrary

astronomical images, and it hands back the astrometric solution. I've not tried it for images as wide-field as you have, but I suspect it will work just fine. See <http://astrometry.net/> for more info.

- Marshall

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Subject: Re: Least square fitting  
Posted by [Bringfried Stecklum](#) on Wed, 21 May 2008 07:47:53 GMT  
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Marshall Perrin wrote:

> MichaelT <[michael.theusner@googlemail.com](mailto:michael.theusner@googlemail.com)> wrote:

>> Hi all,

>>

>> I have a problem with a rather complicated function depending on four  
>> parameters which I try to find using least square fitting and I don't  
>> know exactly how to do it.

>>

>> The basic problem is the following:

>> I have an astronomical image of a star field and try to relate the sky  
>> coordinates (right ascension, declination) of the stars to the pixel  
>> coordinates (x, y).

>

> So if you truly wish to know how to properly solve this particular  
> mathematical problem, I will leave that to other wiser experts to  
> answer.

>

> If, however, what you really want is just to know the astrometric  
> solution for your data, you may be interested to know about  
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> collaborators which intends to solve this particular problem once and  
> for all, for everyone. You feed their web service arbitrary  
> astronomical images, and it hands back the astrometric solution. I've  
> not tried it for images as wide-field as you have, but I suspect it  
> will work just fine. See <http://astrometry.net/> for more info.

>

> - Marshall

I agree with Marshall. Astrometry.net offers a good service and the code works well if the FOV is not too small (i.e. there are enough stars to match). However, it does not take (yet) advantage of the WCS info provided by the FITS header. For your purpose the TERAPIX procedures SCAMP and SWARP can be used. The former establishes the astrometry (including field distortion) while the latter does the image transformation, i.e. creates a non-distorted output mosaic. You might want to have a look at the TERAPIX website for more information.

Regards,

Bringfried

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Subject: Re: Least square fitting  
Posted by [MichaelT](#) on Wed, 21 May 2008 08:45:56 GMT  
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Thanks Marshall and Bringfried,

I will try it out. Actually, I really only need to know the conversion parameters. Everything else can then be done using those.

It would certainly be nice to do it all yourself, but, selecting the reference stars is always a hassle.

Michael

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