
Subject: DERIVSIG derivation problems

Posted by [asdf](#) on Mon, 23 May 2011 16:20:30 GMT

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Hi,

Looking through the code for DERIVSIG, I'm a little confused about its derivation.

From DERIV, it uses a 3-point Lagrange interpolant, so (as is in the code), for inner points (ignoring end points for now)

$$df/dx = y_0(2x-x_1-x_2)/(x_{01}*x_{02}) + y_1(2x-x_0-x_2)/(x_{10}*x_{12}) + y_2(2x-x_0-x_1)/(x_{20}*x_{21})$$

; Where: $x_{01} = x_0 - x_1$, $x_{02} = x_0 - x_2$, $x_{12} = x_1 - x_2$, etc.

In IDL, this is how it is written (from DERIV line 69-71,74-76):

```
x12 = x - shift(x,-1) ;x1 - x2
x01 = shift(x,1) - x ;x0 - x1
x02 = shift(x,1) - shift(x,-1) ;x0 - x2
d = shift(y,1) * (x12 / (x01*x02)) + $ ;Middle points
  y * (1./x12 - 1./x01) - $
  shift(y,-1) * (x01 / (x02 * x12))
```

If I do a standard error propagation formula

$(\sigma^2 = \sum(\sigma_i^2 * (\frac{\partial f}{\partial x_i}), i))$, then I get (in the terms defined above)

$$\sigma_{dfdx} = \sqrt{(2x-x_1-x_2)^2/(x_{01}*x_{02})^2*\sigma_{y0}^2 + (2x-x_0-x_2)^2/(x_{10}*x_{12})^2*\sigma_{y1}^2 + (2x-x_0-x_1)^2/(x_{20}*x_{21})^2*\sigma_{y2}^2)}$$

In IDL, making use that $x=x_1$, and since all the x_{ij} terms are squared in the error propagation formula, we set $x_{ij}=x_{ji}$, I would get:

```
x01=shift(x,1)-x
x02=shift(x,1)-shift(x,-1)
x12=x-shift(x,-1)
sig_y0sqr=shift(sigy,1)*shift(sigy,1)
sig_y1sqr=sigy*sigy
sig_y2sqr=shift(sigy,-1)*shift(sigy,-1)
sigd=(x12^2/(x01*x02)^2*sig_y0sqr + (2*x-shift(x,-1)-shift(x,1))^2/
(x01*x12)^2*sig_y1sqr + x01^2/(x02*x12)^2*sig_y2sqr)
```

Yet in DERIVSIG, this is the equation used:

```
dsq=shift(x,-1)-shift(x,1)
dsq=dsq*dsq
dy=shift(y,-1)-shift(y,1)
sigd=(shift(sigy,-1)*shift(sigy,-1) + shift(sigy,1)*shift(sigy,1))/dsq
sigd=sqrt(sigd)
```

In the notation I used above, this is equivalent to saying:

$\text{sigd} = \sqrt{1./x20^2 \cdot \text{sig_y0}^2 + 1./x20^2 \cdot \text{sig_y2}^2}$

Working through how this compares with the equation I have above, the only way I could get them to match is if DERVSIG is ASSUMING that $x1 - x0 = x2 - x1$, or that the spacing is uniform (not unity, but equal on each side of a point). This appears to be incorrect (the documentation certainly doesn't mention that the spacing needs to be uniform). Have I missed something somewhere?

Michael
