Subject: Multiplying very high with very low numbers: erfc * exp
Posted by tho.siegert on Thu, 03 Apr 2014 09:35:10 GMT
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Hello,
for my MCMC fitting program, I need to evaluate functions of the form (Gaussian with a one sided exponential tail towards lower x-values):

f(a,b,c,d) * erfc(g(a,b,c,d)) * exp(h(a,b,c,d)) := X * Y * Z = F

where f,g and h are certain functions of the parameters a,b,c and d.

It almost always happens that the numbers of these three factors are like:

F = X * Y * Z = 1e2 * 1e-999 * 1e1000 = 1e3

Which is a big problem since 1e-999 is represented as 0 and 1e1000 is represented as infinity, thus the result being 0, infinity or nan, but definetly not 1e3.
As a work-around, I went to log-space such that:

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F = exp( ln(F) ) = exp( ln(X * Y * Z) ) = exp( ln(X) + ln(Y) + ln(Z) ) =
  = exp( ln(f(a,b,c,d)) + ln(erfc(g(a,b,c,d))) + ln(exp(h(a,b,c,d))) ) :=
 := exp(      Q       +        W          +         E        )
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Q and E are no problem to evaluate since f() is just a rational function and ln(exp(h())) is just h().
However, W = ln(erfc(g())) contains the same problem as above. If g() is far negative from 0, erfc(g()) is just 2 (and not e.g. 2 - 1e-99). If g() is far positive from 0, erfc(g()) is just 0, returning W as -Inf (as erfc(g()) should actually be something like 1e-99).

Now, I looked up several representations of the erfc() function in order to build something like a lnerfc - function. I have chosen the erfcc() function in Numerical recipes, Chapter 6, Special Functions (around page 214) which is also given in Wikipedia at
http://en.wikipedia.org/wiki/Error_function#Numerical_approx imation
This approximation has two major advantages:
1) It is represented as proprotional to an exponential function, for which the ln can easily be calculated.
2) The fractional error is "everywhere less than 1.2e-7".

Including all these work-arounds, F = X * Y * Z can be calculated to a good enough precision (for me).

However (again), as you might already think of, it takes a while to calculate F. In a MCMC run, this function has to be evaluated over and over again. If there is more than one such a function present in my data (say N), I need to fit, i.e. evaluate something like:

sum(F_i, i=0..N)

over and over again (typically N = 20..30).

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I hope you understand the problem and are not overwhelmed by this wall of text.
I appreciate any suggestions.

Cheers,
Thomas

Subject: Re: Multiplying very high with very low numbers: erfc * exp
Posted by lecacheux.alain on Thu, 03 Apr 2014 13:21:31 GMT
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You might for instance consider Rational Chebyshev approximations of X*Y, which are often used for computing the "erfcx" function (i.e. exp(x^2)*erfc(x)), whose shape is not far from the one you are dealing with.
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Subject: Re: Multiplying very high with very low numbers: erfc * exp
Posted by tho.siegert on Wed, 16 Apr 2014 12:09:10 GMT
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Okay, since I already actually had a lnerfc function, but was too silly to make it work properly, i post my solution:

```
function lnerfc, x, y

  a = [-1.26551223d, 1.00002368d, 0.37409196d, 0.09678418d, -0.18628806d, 0.27886807d,
-1.13520398d, 1.48851587d, -0.82215223d, 0.17087277d]

  t = 1d / (1d + 0.5d * abs(x))

  tau = t * exp( -x*x + (a[0] + t * (a[1] + t * (a[2] + t * (a[3] + t * (a[4] + t * (a[5] + t * (a[6] + t * (a[7] + t
* (a[8] + t * a[9]))))))))))

  y = alog(t) + ( -x*x + (a[0] + t * (a[1] + t * (a[2] + t * (a[3] + t * (a[4] + t * (a[5] + t * (a[6] + t * (a[7] +
t * (a[8] + t * a[9]))))))))))

  lt0 = where(x lt 0d,/null)

  y[lt0] = y[lt0] + alog(2d / tau - 1d)

  return, y

end
```

It is again taken from Numerical recipes, Chapter 6.2, Special Functions, just translated to logarithm space. This is indeed based on Chebyshev fitting. Thanks alx!

Regards,
Thomas