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Subject: FFT and Parseval

Posted by [baptiste.cecconi](#) on Mon, 28 Apr 2014 10:58:21 GMT

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Dear IDL guys,

I recently tried to check the conservation of energy (Parseval's theorem) through the IDL implementation of FFT, and I came to a somewhat surprising result.

Here a sample code that shows my point:

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```
N=1000
```

```
x = randomn(0,N) ; random series of data with 1000 elements
```

```
fft0 = fft(x,-1) ; fourier transform (to freq domain) of x
```

```
print,total(x^2.) ; total energy of the signal in time domain
```

```
print,total(abs(fft0)^2.)/N; total energy of the signal in freq domain (according to Parseval's theorem)
```

```
fft1 = fft(x,1) ; inverse fourier transform (freq to time domain) of x
```

```
print,total(abs(fft1)^2.)/N; total energy of the signal in freq domain (using inverse fft)
```

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From this little code, it is clear that

(1)  $\text{total}(x^2.) = \text{total}(\text{abs}(\text{fft0})^2.) * N$

(2)  $\text{total}(x^2.) = \text{total}(\text{abs}(\text{fft1})^2.) / N$

While quation (2) is fully consistent with Parseval's equation, (1) is not, by a  $N^2$  factor.

In the IDL documentation, it is stated that "A normalization factor of  $1/N$ , where  $N$  is the number of points, is applied during the forward transform." However, I'm not sure this solves anything here.

I have some difficulties to convince myself that the direct FFT transform is invoked with a negative "direction" parameter (as stated in IDL documentation).

Parseval theorem is recalled here:

[http://en.wikipedia.org/wiki/Parseval's\\_theorem](http://en.wikipedia.org/wiki/Parseval's_theorem) (see DFT equation)

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Subject: Re: FFT and Parseval

Posted by [Moritz Fischer](#) on Mon, 28 Apr 2014 11:30:04 GMT

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This all about the conventions used when defining the Fourier transform:

- sometimes it's multiplied by a factor of  $1/n$  in the forward

transformation, and not in the inverse ( as in the IDL forward transformation )

- sometimes it's scaled in the inverse transformation ( as in the wikipedia definition of the DFT following the link you gave)
- and I as a mathematician prefer the scaling by  $1/\sqrt{n}$  in both forward and inverse transformation, because a) its symmetric and b) there won't be a factor in the formulation of the \*conservation\* ( not scaling ... ) of energy.

In either of the above cases you get

$x = \text{FFT}(\text{FFT}(x,-1),1)$

but comparing the energies of time and spectrum you have to compensate the  $1/\sqrt{n}$  of the applied  $1/n$  scaling factor, and your (1) reads:

$$\text{total}(\text{abs}(x)^2) = \text{total}(\text{abs}(\text{fft0} * \sqrt{N})^2)$$

I guess the scaling convention in IDL is chosen for performance reasons.

And note that the direction parameter is the sign of the argument of  $\exp(\cdot)$ , i.e. negative for forward transformation, by most conventions.

Am 28.04.2014 12:58, schrieb baptiste.cecconi@obspm.fr:

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>  $\text{total}(\text{abs}(\text{fft1})^2) / N$   
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