Subject: Solving system of ODEs backwards in time?
Posted by BLesht on Sun, 30 Jul 2017 18:37:45 GMT
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I have a system of ODEs describing how a system with N state variables (C) evolves in time. The basic equation set is dC/dt = (W + A dot C) / V in which C is the state variable vector at time i, V is a vector of constants, W is a known vector (a function of t), and A is a known matrix (similarly time variable). Given an initial condition C[0], I've been using LSODE to solve for the successive time steps, updating the initial condition and values of W and A along the way. This has worked well.

Now I'd like to reverse the problem. That is, if I know the value of the state vector at time i, and the values of W and A at time i-1, I'd like to compute the value of the state vector at time i-1. In essence, I want to know what the initial condition had to be to arrive at the current state of the system given known V, W and A.

Frankly, it's been many, many years since I took an ODE class and I wasn't very adept then. I'd greatly appreciate any advice on how to approach this problem.

Thanks, Barry

Subject: Re: Solving system of ODEs backwards in time?
Posted by Markus Schmassmann on Wed, 02 Aug 2017 14:58:32 GMT
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On 07/30/2017 08:37 PM, Barry Lesht wrote:
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> variables (C) evolves in time. The basic equation set is dC/dt = (W
> + A dot C) / V in which C is the state variable vector at time i, V
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> wasn't very adept then. I'd greatly appreciate any advice on how to
> approach this problem.

; example inputs
n=10

```
tt=100
  w=randomu(seed,n,tt-1,/double)
  a=randomu(seed,n,n,tt-1,/double)
  v=randomu(seed,n,/double)*100
c0=randomu(seed,n,/double)

; run it forward
ca=dblarr(n,tt)
ca[*,0]=c0
for i=0,tt-2 do ca[*,i+1]=ca[*,i]+(w[*,i]+a[*,*,i]#ca[*,i])/v

; run it back
cb=dblarr(n,tt)
cb[*,-1]=ca[*,-1]
diag_v=dblarr(n,n)
diag_v[lindgen(n),lindgen(n)]=v
for i=tt-2,0,-1 do cb[*,i]=invert(diag_v+a[*,*,i])#(cb[*,i+1]*v-w[*,i])

; compare results
print, ca[*,0]
print, cb[*,0]
```

however, if diag_v+A+a[*,*,i] can't be inverted you get nonsense as result.
So check it by running the inversion forward again

I hope this helps, Markus

Subject: Re: Solving system of ODEs backwards in time?
Posted by BLesht on Wed, 02 Aug 2017 23:50:33 GMT
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Markus - thank you very much! That appears to be exactly what I was looking for. It will take me a bit to adapt it to my model, but I will let you know if it works.

I very much appreciate your help.

Barry

Subject: Re: Solving system of ODEs backwards in time?
Posted by Craig Markwardt on Fri, 04 Aug 2017 15:25:04 GMT
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On Sunday, July 30, 2017 at 2:37:48 PM UTC-4, Barry Lesht wrote:
> I have a system of ODEs describing how a system with N state variables (C) evolves in time. The basic equation set is dC/dt = (W + A dot C) / V in which C is the state variable vector at time i, V is a vector of constants, W is a known vector (a function of t), and A is a known matrix (similarly time variable). Given an initial condition C[0], I've been using LSODE to solve for the successive time steps, updating the initial condition and values of W and A along the way. This has worked well.
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> Frankly, it's been many, many years since I took an ODE class and I wasn't very adept then. I'd greatly appreciate any advice on how to approach this problem.
>
> Thanks, Barry

Why can't you just use a negative time step in your call to LSODE?

Subject: Re: Solving system of ODEs backwards in time?
Posted by BLesht on Fri, 04 Aug 2017 20:01:10 GMT
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Hi Craig,

I'm sorry to seem dense, but I don't see how that applies. Perhaps I haven't explained my problem sufficiently, or perhaps I really don't understand the nuances, or maybe I've been misapplying LSODE (or all the above).

I have a system of 19 coupled ODEs. Let C be the 19 element vector representing the state of the system at time point i. The vector of derivatives is dC(i)/dt = (W(i) + A(i) dot C(i)) / V in which W is a 19-element vector that changes at every point i, A is a 19x19 "transfer" matrix expressing the couplings among the state variables (many zeros) but which also changes at every point i, and V is a 19-element constant vector. Given an initial condition C0, I have been been using LSODE is advance the solution from time i to time i+1 (calculating C(i+1) using a time step of i/4. I repeated those steps for the desired number of i steps.

This seems to work (at least provides answers that agree well with observations) going forward. What I want to do now is start with a known state at time i, and sets of known W vectors and A matrices for times i-1, i-2, ... i-n and find what C(i-n) would have had to be to result in the observed C(i) given that set of W vectors and A matrices.

What confused me when I was trying to set this up myself was that the state at time i, depends on both the state at time i-1 and the derivatives based on the state at time i-1. That is, the derivative at time i-1 can't be computed without knowing the state at time i-1 because of the A dot C term.

Thanks, Barry

Subject: Re: Solving system of ODEs backwards in time?
Posted by Craig Markwardt on Fri, 04 Aug 2017 21:33:05 GMT
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On Friday, August 4, 2017 at 4:01:13 PM UTC-4, Barry Lesht wrote:
...
> This seems to work (at least provides answers that agree well with observations) going forward. What I want to do now is start with a known state at time i, and sets of known W vectors and A matrices for times i-1, i-2, ... i-n and find what C(i-n) would have had to be to result in the observed C(i) given that set of W vectors and A matrices.

I'm just saying, LSODE takes a "step" parameter, H, and that parameter can be negative as well as positive. It's just as easy to integrate backward in time as it is to integrate forward in time.

Craig
